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On the arrangement of Commutation, or D *and* N, *Tables. By* MR. JAMES CHISHOLM, *of the North British and Mercantile Insurance Company, Edinburgh.*

[Read before the Institute, 27th January, 1868.]

THERE has been no more valuable improvement in the science of Assurance calculations than that effected by the invention of the Commutation or D and N System of calculation;—a method which, as is now universally known through the writings of Baily, Professor De Morgan and Mr. Gray, we owe, in this country, to the genius of Mr. George Barrett, of Petworth, Sussex. It appeared at first in an incomplete and rather inconvenient form, but various important modifications and extensions of the principle of the method were some time afterwards made; and it was practically applied to the calculation of Life Contingencies by the late eminent actuary Mr. Griffith Davies, with whose name it is now generally associated.

Several works containing voluminous Tables on the Commutation principle have since been published, and actuarial calculations are now greatly facilitated by their use. A certain degree of confusion, however, has nearly from the first surrounded the system, which is especially perplexing to the student, and has, no doubt, deterred many from using the formulæ to that extent which their importance and convenience would warrant.

This confusion has arisen, partly, but to a very slight extent, from a difference between Mr. Davies and Professor De Morgan in tabulating the Assurance values in Column C, and partly from the change introduced by Dr. Farr in 1844 in the system of summation of the columns, or from both causes combined.

The change referred to was from the Terminal to the Initial System of summation; using the latter phrase, as proposed by Professor De Morgan in a paper in Volume X. of the *Journal* "On the Forms under which Barrett's method is presented, &c.," to denote the plan introduced by Dr. Farr; and the former for that followed by Davies in summing the fundamental columns.

On a comparison of the formulæ for the values of an Annuity and an Assurance on these two systems, a want of symmetry is at once apparent. On the Initial System they are $\frac{N_{x+1}}{\mathrm{D}_x}$[*] and $\frac{\mathbb{M}_x}{\mathrm{D}_x}$, the former signifying the value of an Annuity of which the first payment is due at the *end* of a year, the latter the value of an Assurance payable at the *end* of the year of death; while Davies's formulæ for the same values are $\frac{N_x}{D_x}$ and $\frac{M_x}{D_x}$ respectively.

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[*] An explanation of the styles of capital employed in this paper will be found in the postscript.

If Davies's formulæ are right, we would naturally expect them, when transferred to the Initial System, to appear as $\frac{N_{x+1}}{D_x}$ and $\frac{M_{x+1}}{D_x}$; and this consideration points us to the M column, or to the C column from which it proceeded, as the places where we are likely to find that the want of symmetry has arisen.

We propose, in this paper, to inquire generally into the causes of the confusion already alluded to, and in doing this it is hoped that some part of it will be cleared away.

The most satisfactory mode of proceeding will be to construct a Commutation Table, on proper principles, as a basis of comparison, to note the differences between it and the Tables of Davies and Dr. Farr, and to examine whether they are essential, and whether they relate to the principle of construction, or not.

At the outset, however, we are led to ask if our Tables of Mortality, on which of course the D and N Tables are founded, are properly constructed; and to this point, therefore, we shall first turn our attention.

In tabulating the numbers living and dying in a Table of Mortality, the practice is to put the number dying in any year of life in a line with the number out of whom they die,—that is, opposite the last age which they have attained. For instance, opposite or on a line with x, l_x signifies the number alive at age x, and $d_x (= l_x - l_{x+1})$ the number dying out of the l_x in the year of life after age x, or in the $(x+1)$th year of age.

Now, on consideration, it will be acknowledged, we think, that it would be more correct to put d_x, the number who die in the $(x+1)$th year of age, opposite age $x+1$. For the d_x and l_{x+1} may be conceived to have sprung from l_x as their origin; both having attained age x, the former *die in* their $(x+1)$th year, the latter *live over* it. The Table, in fact, is merely a list of the numbers of those who, out of a certain initial number, *go forward* to *die in* and *live over* each successive year of life. In this view of the case, therefore, the numbers that have passed a particular age naturally go together, and are accounted for or registered at the next higher age as having *died in* or *lived over* the preceding year.

The case is still stronger when it is considered that for all purposes of calculation, unless otherwise expressed, the $(l_x - l_{x+1})$

deaths are held to have taken place at the *end* of the $(x+1)$th year of life; the Assurance claims arising under them are then payable, and the deaths may thus be held to have occurred at age $x+1$, and, consequently, may be symbolised at that age by δ_{x+1}.

According to this plan of formation the Table would stand thus:—

Carlisle Mortality.

Age.	No. alive.	No. dying in the preceding year.
x.	$\Sigma\delta_{x+1}=l_x$.	δ_x.
0	10000	
1	8461	1539
2	7779	682
3	7274	505
4	6998	276
. .	. . . .	. . .
101	7	2
102	5	2
103	3	2
104	1	2
105		1

where $l_x=\delta_{x+1}+l_{x+1}$ and $\Sigma\delta_{x+1}=\delta_{x+1}+\delta_{x+2}+\delta_{x+3}+\&c.=l_x$.

The Greek letter δ is adopted to represent the number of deaths in this arrangement of the Table to prevent confusion.

The idea of tabulating d_x as now proposed seems to have been in the mind of Griffith Davies, when, in his *Treatise on Annuities* (p. 94) he says:—

"By the Table * * * * we find that at the town of "Northampton, from 1735 to 1780, out of 11650 children all "born alive,

"8650 attained the age of 1 year, and 3000 died during the 1st year
"7283 . . . . . 2 years, . 1367 . . . . 2nd . .
"6781 . . . . . 3 . . . 502 . . . . 3rd . .
"6446 . . . . . 4 . . . 335 . . . . 4th . .

"and so on, as in the Register known by the name of the North-"ampton Table."—and then follows the Table as ordinarily constructed.

Further on (page 238), the same arrangement as is given above is repeated; and, at the same place, in explaining the construction of the C column, or rather the P column as he called it, he says:—

"Again, referring to the column expressing the number of "claims to be made on the supposed fund at the end of the 1st, "2nd, 3rd, &c. year, it is manifest that if only £1 were paid to

" each claimant, the present value of the claims becoming due at " the end of the

" 1st year would be 3000 × ·970874 = 2912·622
" 2nd . . . . 1367 × ·942596 = 1288·529
" 3rd . . . . 502 × ·915142 = 459·401
" 4th . . . . 335 × ·883487 = 297·643

" and so on as in column P of the subjoined Table." In this column P (see p. 239) the value 2912·622 is put opposite age 1, 1288·529 opposite age 2, and so with the other values.

As shown above, then, we see that in *some* instances in his work, Davies arranged the numbers of living and dying as now suggested. But whether this was his uniform practice, is not of any consequence. It is enough that in constructing column P, he acted on the footing of this arrangement of the Mortality Table. It will be of importance to bear this in mind when his form of Commutation Table comes to be considered.

The same idea of tabulating d_x is expressed in Jones (*Treatise on Annuities,* Arts. 189 and 192) when he calls the number opposite age x in the column of decrements in a Mortality Table d_{x+1}, indicating that for the purpose at least for which he then uses the number, it would have been more suitably placed at age $x+1$.

We are now in a position to go on with the construction of a Commutation Table; and in doing so we shall adopt the Initial System of summation, it being, as appears to us, the most natural one, and consequently the best for the purpose for which the Table is to be used.

There is nothing unusual in the formation of the Annuity columns. In the Mortality Table we have at age 0 the number l_0, and this multiplied by v^0 gives the first value in the D column, which we place at age 0 and call D_0. Thus $D_0=l_0v^0$, $D_1=l_1v^1$, $D_2=l_2v^2$, &c., as appears in all Commutation Tables.

The N column is thus composed:—

$N_0=D_0+D_1+D_2+$&c. to the end of life.
$N_1=D_1+D_2+D_3+$&c. do.

and similarly with the S column,

$S_0=N_0+N_1+N_2+$&c. to the end of life.
$S_1=N_1+N_2+N_3+$&c. do.

The fundamental Assurance column C is not the same as in most of the other Tables, and special attention must therefore be given to its formation. It is formed in the same way from the

Mortality Table. Thus the first number in the column of decrements occurs at age 1. We accordingly multiply δ_1 by v^1 and get the first value in the C column, which we naturally place at age 1. We have then $C_0=0$, $C_1=\delta_1 v^1$, $C_2=\delta_2 v^2$, and so on.

The M and R columns are constructed from C and M respectively, just as the corresponding Annuity columns N and S were formed from D and N:

$$M_1 = C_1 + C_2 + C_3 + \&c. \text{ to the end of life.}$$
$$M_2 = C_2 + C_3 + C_4 + \&c. \qquad \text{do.}$$

and

$$R_1 = M_1 + M_2 + M_3 + \&c. \qquad \text{do.}$$
$$R_2 = M_2 + M_3 + M_4 + \&c. \qquad \text{do.}$$

For the sake of clearness, the following Specimen Table is appended:—

Commutation Table for Single Lives. Carlisle 3 *per cent.*

TABLE I.—*Initial arrangement.*

Age.	$l_x \times v^x$	$\Sigma D_x = D_x + D_{x+1} + \&c.$	$\Sigma N_x = N_x + N_{x+1} + \&c.$	$\delta_x \times v^x$	$\Sigma C_x = C_x + C_{x+1} + \&c.$	$\Sigma M_x = M_x + M_{x+1} + \&c.$
x	D_x	N_x	S_x	C_x	M_x	R_x
0	10000·0000	183198·2348	3885247·9466			
1	8214·5631	173198·2348	3702049·7118	1494·1750	4664·1296	70035·6729
2	7332·4536	164983·6717	3528851·4770	642·8506	3169·9546	65371·5433
3	6656·7404	157651·2181	3363867·8053	462·1465	2527·1040	62201·5887
4	6217·6324	150994·4777	3206216·5872	245·2225	2064·9575	59674·4847
⋮	⋮	⋮	⋮	⋮	⋮	⋮
101	·3536	·7879	1·4576	·1010	·4317	1·1772
102	·2452	·4343	·6696	·0981	·3307	·7455
103	·1429	·1891	·2353	·0952	·2326	·4148
104	·0462	·0462	·0462	·0925	·1373	·1822
105				·0449	·0449	·0449

The Expression here for the value of an Assurance payable at the *end of the year of death* will be $\frac{M_{x+1}}{D_x}$, just as $\frac{N_{x+1}}{D_x}$ expresses the value of an Annuity payable at the *end of each year of life.*

These expressions are symmetrical, and, if it be admitted that the Table from which they are taken is properly constructed, the analogy between the Annuity and Assurance formulæ may be held to be established.

It may be remarked that the analogy does not hold throughout. There is no corresponding Assurance value to that of an Annuity-due; or, if it were possible to deal practically with such values, the formula deduced for them would not be symmetrical with the Annuity formula.

But this is an erroneous argument, inasmuch as the use of the word Annuity in this sense is improper. Interpreted, an Annuity-due of £1 means a *present payment* of £1, and an Annuity—(the only legitimate sense in which the word can be used)—of £1 thereafter, and the phrase has only been adopted for the sake of convenience, as such values as it indicates are often required. (*On this point see Prof. De Morgan's paper in Vol. X., pp.* 301 *and* 302.)

In comparing the Table we have now constructed with the others in existence, the first that presents itself for consideration is Davies's.

His Table is as below :—

Commutation Table for Single Lives. Carlisle 3 *per cent.*

TABLE II.—*Davies's or Terminal Arrangement.*

Age.	$l_x \times v^x$	ΣD_{x+1} $= D_{x+1} + D_{x+2} +$ &c.	ΣN_x $= N_x + N_{x+1} +$ &c.	$\delta_x \times v^x$	ΣC_{x+1} $= C_{x+1} + C_{x+2} +$ &c.	ΣM_x $= M_x + M_{x+1}$ + &c.
x	D_x	N_x	S_x	C_x	M_x	R_x
		[183198·2348]	[3885247·9466]			
0	10000·0000	173198·2348	3702049·7118		4664·1296	70035·6729
1	8214·5631	164983·6717	3528851·4770	1494·1750	3169·9546	65371·5433
2	7332·4536	157651·2181	3363867·8053	642·8506	2527·1040	62201·5887
3	6656·7404	150994·4777	3206216·5872	462·1465	2064·9575	59674·4847
4	6217·6324	144776·8453	3055222·1095	245·2225	1819·7350	57609·5272
⋮	⋮	⋮	⋮	⋮	⋮	⋮
101	·3536	·4343	·6696	·1010	·3307	·7455
102	·2452	·1891	·2353	·0981	·2326	·4148
103	·1429	·0462	·0462	·0952	·1373	·1822
104	·0462			·0925	·0449	·0449
105				·0449		

We here observe that the fundamental columns D and C occupy the same positions as in Table I., and, bearing in mind how, in that Table, these were formed from the numbers in the Mortality Table, and remembering also what was before stated, that Davies acted on the understanding that the Mortality Table was in the same form as the one on which we founded in constructing Table I., this is just what we should have expected.

The only difference, then, between Tables I. and II. is this, that in the latter, the N, S and M, R columns are raised one step, whereas in the former, the values at the youngest age in the derived columns are put in a line with the values at the youngest age in the fundamental columns; or, in other words, Davies has adopted the Terminal System of summation, while the Initial is used in Table I. This does not affect the principle of construction, and we consequently find that the expressions for the value of an Annuity and of an Assurance are symmetrical, as they were before, the one being $\frac{N_x}{D_x}$, and the other $\frac{M_x}{D_x}$.

Jones in his *Treatise on Annuities* followed Davies exactly in his arrangement of the columns, but it is easy to miss noticing this owing to his not having stated specifically how he derived column M from column C, and to his not having printed the latter along with the other columns.

In Art. 116, however, the following explanation is given:—

> "Column M is formed by multiplying the decrements opposite each age "in Table I. [Table of Mortality] by the present value of £1 due the same "number of years as the age increased by unity, and taking the successive "sums from the extremity of life, as in the formation of column N from "the numbers in column D."

Paying attention to the statement here that column M is formed "*as in the formation of column* N from the numbers in column D," we must agree that column M would have appeared *one step above* column C if the latter had been printed; and, noting the position M occupies in Jones's Commutation Table, we find that the first value in C must have been at age 1.

Again, in deducing the value of an Assurance by Davies's method (Art. 189) he brings out the formula

$$A_x = \frac{vd_{x+1} + v^2d_{x+2} + v^3d_{x+3} + \&c.}{l_x}$$

or

$$= \frac{v^{x+1}d_{x+1} + v^{x+2}d_{x+2} + v^{x+3}d_{x+3} + \&c.}{v^x l_x}$$

calling it equal to $\frac{M_x}{D_x}$, and by so doing, he shows that the M column has been raised above the C column one step. For, independently of what has been shown above, it is *most natural* to suppose that he would place $v^{x+1}d_{x+1}$ opposite age $x+1$ in column C, in which case the expression for the value of an Assurance would have been $\frac{M_{x+1}}{D_x}$ had the M column not been raised.

Intimately connected with the labours of Griffith Davies are those of Professor De Morgan, who gave to the profession two valuable papers which first appeared in the *Companion to the Almanac* for 1840 and 1842.

With the exception of the C column, the Commutation Table there given is the same as Davies's, and it is not necessary to reproduce it here. The exception, however, is one which we must notice. Professor De Morgan makes $(l_0-l_1)v=\mathbb{C}_0$, $(l_1-l_2)v^2=\mathbb{C}_1$, &c., that is to say, he raises the C column one step higher than it is in Davies's arrangement or in Table I. But he did not alter the position of the M and R columns; and these, as well as the N and S columns are in the same position relatively to each other and to D as with Davies.

The only effects of the alteration are that the C column occupies what we venture to think is an anomalous position, and that the Table presents the rather curious condition of having the Annuity division summed on the Terminal and the Assurance division on the Initial System. The formulæ themselves are affected only to a very trifling extent. The difference between them and Davies's formulæ appears only in Benefits on the C column, and this is so very little required that no confusion in using Professor De Morgan's papers is practically experienced.

It may be interesting, however, as well as useful, to append some of the "formulæ and additional notation" given by him in these papers, making the necessary corrections on the supposition that column C is placed as Davies had it.

The following may be compared with those given on page 333 of Vol. XII of this *Journal*:—*

I. $N_x=D_{x+1}+D_{x+2}+D_{x+3}+\ldots.$ to the end of life.
$M_x=C_{x+1}+C_{x+2}+C_{x+3}+\ldots.$ do.
$S_x=N_x+N_{x+1}+N_{x+2}+\ldots.=D_{x+1}+2D_{x+2}+3D_{x+3}+\ldots.$
$R_x=M_x+M_{x+1}+M_{x+2}+\ldots.=C_{x+1}+2C_{x+2}+3C_{x+3}+\ldots.$

II. $N_x-N_{x+1}=D_{x+1}$ $S_x-S_{x+1}=N_x$
$M_x-M_{x+1}=C_{x+1}$ $R_x-R_{x+1}=M_x$

* * * * * *

V. $D_x+\ldots.+D_y=N_{x-1,\,y-x+1}$
$C_x+\ldots.+C_y=M_{x-1,\,y-x+1}$
$D_x+\ldots.+(y-x+1)D_y=S_{x-1,\,y-x+2}$
$C_x+\ldots.+(y-x+1)C_y=R_{x-1,\,y-x+2}$

* * * * * *

VIII. $C_x=vD_{x-1}-D_x$
$M_x=vN_{x-1}-N_x$
$R_x=vS_{x-1}-S_x$

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* Professor De Morgan's papers have been reprinted in this *Journal* in Volume XII., p. 328, and Vol. XIII., p. 129. Mr. Gray's papers, afterwards referred to, are reprinted in Volume X.

It should be observed that the preceding remarks apply also to the papers by Mr. Gray which originally appeared in the Mechanics' Magazine for 1842, and which are intended as an Introduction to those of Professor De Morgan.

There only remains now to be considered the Table of Dr. Farr. For the purpose of comparison the following Specimen of his arrangement is annexed.

Commutation Table for Single Lives. Carlisle 3 per cent.

TABLE III.—*Dr. Farr's arrangement.*

Age.	$l_x \times v^x$	$\Sigma D_x = D_x + D_{x+1} + \&c.$	$\Sigma N_x = N_x + N_{x+1} + \&c.$	$d_x \times v^{x+1}$	$\Sigma \mathfrak{C}_x = \mathfrak{C}_x + \mathfrak{C}_{x+1} + \&c.$	$\Sigma \mathfrak{M}_x = \mathfrak{M}_x + \mathfrak{M}_{x+1} + \&c.$
	D_x	N_x	S_x	$\mathfrak{C}_x$	$\mathfrak{M}_x$	$\mathfrak{R}_x$
0	10000·0000	183198·2348	3885247·9466	1494·1750	4664·1296	70035·6729
1	8214·5631	173198·2348	3702049·7118	642·8506	3169·9546	65371·5433
2	7332·4536	164983·6717	3528851·4770	462·1465	2527·1040	62201·5887
3	6656·7404	157651·2181	3363867·8053	245·2225	2064·9575	59674·4847
4	6217·6324	150994·4777	3206216·5872	173·3844	1819·7350	57609·5272
: :	: : : :	: : : : :	: : : : :	: : : :	: : : :	: : : : :
101	·3536	·7879	1·4576	·0981	·3307	·7455
102	·2452	·4343	·6696	·0952	·2326	·4148
103	·1429	·1891	·2353	·0925	·1373	·1822
104	·0462	·0462	·0462	·0449	·0449	·0449
105						

By this Table the expression for the value of an Assurance is $\frac{\mathfrak{M}_x}{D_x}$, and of an Annuity $\frac{N_{x+1}}{D_x}$, as pointed out in the beginning of this paper. These, unlike the same formulæ under Tables I. and II., are not symmetrical, and from what has been already said the reason of this will be evident.

When Dr. Farr introduced the change from the Terminal to the Initial System of summation,* he adopted Professor De Morgan's $\mathfrak{C}$ column, and with it the whole of his Assurance division, which, as was before stated, was already summed on the Initial principle, and did not therefore require any change. The only alteration on Professor De Morgan's Table was that columns N and S were brought down to a level with D. The change thus affected only the Annuity half of the Table, and this accounts for the want of symmetry in Dr. Farr's formulæ.

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* See Part II. of the Appendix to the Registrar-General's Report for 1844, pages 526 and 527 and 592 and 593. Compare also Introduction to the English Life Table No. 3, page 119 *et seq.*

Besides the confusion, then, of having two different systems in existence as the basis of our formulæ, we have the additional confusion that in the one system (Davies's) the Assurance and Annuity Formulæ are counterparts of each other, while in the other system (Dr. Farr's) they are not; and this disappearance of the analogy between the two divisions of the Table on Dr. Farr's system certainly renders the formulæ more difficult of remembrance.

It should not be forgotten, however, that when Professor De Morgan contributed his papers before referred to, and when Dr. Farr first introduced his change, the posthumous work of Mr. Davies from which we have quoted was not published; and that the Tract by him on Life Contingencies, which was at the time, the principal work on the subject, did not contain so full an exposition of his views as his later Treatise, and was besides very scarce.

On referring to this "Tract" we find that the column P is not tabulated along with the other columns; and that all the explanation that is given of the formation of M is contained in a short paragraph expressed in almost the very words employed by Jones for the same purpose, as quoted at page 206.

It will be seen to be important to quote Davies's own words. He says (p. 36):—

> "The numbers in column M have been deduced by multiplying the "decrements given opposite each age in Table X. [*Table of Mortality*] "by the present value of £1 due as many years hence as are equal to that "age increased by unity; and then, beginning at the extremity of life, and "taking the successive sums of the results, the same as in the formation of "column N from D, or S from N."

It will be observed that Davies directs the successive sums to be taken, "the same as in the formation of column N from D, *or* S *from* N." Now, in forming N from D, that column was raised one step, while S was kept on the same level as N, so that from this passage we could not tell whether Davies meant the column from which M was formed to be one step below, or on a level with M.

In the absence of more precise information, therefore, one would be quite at liberty, while professing to follow Davies, to place the C column in either position. Thus, Professor De Morgan fixed C with the first value at age 0, and this position of the column was confirmed by Dr. Farr; but, as it turned out afterwards, (on the publication of Davies's Treatise by his Executors in 1856) this was not the place Davies meant C to occupy. We have here, then, the explanation of the manner in which C ever came to hold two positions as it does now. The small extent to which Davies's work is used in practice, and the unimportance of the C column in itself, have no doubt contributed to this fact being overlooked, apparently, until now.

To summarise, Davies placed the first value in D at age 0, and the first value in P or C at age 1, and summed these columns on the Terminal System. Professor De Morgan placed the first values in both columns D and C at age 0, summing the former on the Terminal, and the latter on the Initial System. Dr. Farr placed the first values in D and C at age 0 following Professor De Morgan, and summed both columns on the Initial System. And lastly, in the specimen Table constructed for this paper N and M are formed by summing D and C, respectively, on the Initial System; these last-named columns occupying the position assigned to them by Davies.

If it be agreed that this is their proper position, Table I. will present the correct form of a Commutation Table on the Initial, and Table II. the correct form on the Terminal system of summation.

The principal writers who have followed Davies are Jones in his *Treatise on Annuities* (1844), Gray, Smith and Orchard in their *Three per cent Assurance and Annuity Tables* (1851), and Brown in his *Report* on the *Madras Military Fund* (1863). In the two last-named works no reference at all appears to be made to C, so that it is impossible to say whether the authors have followed Davies in the position he gave to the column, or not. Professor De Morgan in his two papers in the *Companion to the Almanac* and Mr. Gray in his Introduction to them may also be said to have followed Davies, for the displacement there of the C column has little practical effect.

Dr. Farr's method has been followed in those of the Registrar-General's Reports since 1844 where the subject has been touched upon, and by Chisholm in his *Commutation Tables* (1858), while other writers in the *Journal of the Institute* and elsewhere have adopted that method which seemed most convenient to them. Such appears to be the position in which we are now placed with reference to the Commutation System. The settlement of the question which form of it is to be adopted as the standard will be a great advantage. Until then, we must make the best of the forms we have got, and endeavour to understand clearly the symbols we have to work with. If the present paper prove at all beneficial in this last respect the object of the writer will have been fully served; and, in conclusion, it may be remarked that it will be found convenient, in using the Tables, to adopt some distinctive lettering for the different forms, such as that shown in the Postscript.

NOTE.—Since the foregoing was written it has been observed that the registration of the numbers living and dying, which occurs at the margin of the "Student's Table," compiled in 1849 by Mr. W. T. Thomson, Manager of the Standard Assurance Company, is made in the same way as suggested in this paper. In the arrangement, also, of *all* the columns Mr. Thomson has exactly followed Davies, and in his work entitled "Actuarial Tables," where the Student's Table is referred to and explained, he has consistently carried out his plans by giving the values of an Assurance at all ages from 1 to 105 in place of from 0 to 104.

POSTSCRIPT.

The following statement is appended in explanation of the different styles of lettering employed in the preceding paper. The columns in Davies's arrangement, as the correct form of Table on the Terminal System, were first lettered; next the columns as in Table I. in the paper, as the correct Initial arrangement, and lastly the columns as in Dr. Farr's arrangement.

Terminal arrangement.

Elements of Column.	Name of Column.
$v^x l_x$	D_x
$D_{x+1} + D_{x+2} + D_{x+3} + \&c.$	N_x
$N_x + N_{x+1} + N_{x+2} + \&c.$	S_x
$v^x(l_{x-1} - l_x) = v^x \delta_x = v^x d_{x-1}$ . . .	C_x
$C_{x+1} + C_{x+2} + C_{x+3} + \&c.$	M_x
$M_x + M_{x+1} + M_{x+2} + \&c.$	R_x

This is the original tabulation of the columns; and Roman Capitals are used for the headings in every case. In Professor De Morgan's modification of this arrangement the heading of the C column will be printed in open capitals; all the other characters as they are above.

Initial arrangement.

Elements of Column.	Name of Column.
$v^x l_x$	D_x
$\mathrm{D}_x+\mathrm{D}_{x+1}+\mathrm{D}_{x+2}+\&c.$	N_x
$N_x+N_{x+1}+N_{x+2}+\&c.$	S_x
$v^x(l_{x-1}-l_x)=v^x\delta_x=v^x d_{x-1}$	C_x
$\mathrm{C}_x+\mathrm{C}_{x+1}+\mathrm{C}_{x+2}+\&c.$	M_x
$M_x+M_{x+1}+M_{x+2}+\&c.$	R_x

In this arrangement, Italic Capitals have been used as the distinctive headings of the columns; but as the D and C columns are the same as Davies's the Roman lettering in these two cases is retained.

Dr. Farr's arrangement.

Elements of Column.	Name of Column.
$v^x l_x$	D_x
$\mathrm{D}_x+\mathrm{D}_{x+1}+\mathrm{D}_{x+2}+\&c.$	N_x
$N_x+N_{x+1}+N_{x+2}+\&c.$	S_x
$v^{x+1}(l_x-l_{x+1})=v^{x+1}\delta_{x+1}=v^{x+1}d_x$	$\mathbb{C}_x$
$\mathbb{C}_x+\mathbb{C}_{x+1}+\mathbb{C}_{x+2}+\&c.$	$\mathbb{M}_x$
$\mathbb{M}_x+\mathbb{M}_{x+1}+\mathbb{M}_{x+2}+\&c.$	$\mathbb{R}_x$

As suggested by Mr. Sprague in last number of the *Journal,* Open Capitals have been used here as the distinctive headings of the columns; but, as the D column is the same as Davies's the Roman lettering for it is still retained. The N and S columns being the same as in the previous arrangement, retain the Italic Capitals as their headings.

Memoir on Instrument for furnishing the D *numbers, to four figures each, in Two Joint Life Annuity Tables, on any basis.*

By JARDINE HENRY, *Author of "The Government Annuity Tables," "The Government Life Annuity Commutation Tables," "The Hand Book for Life Assurers," &c. &c.*

[Read before the Institute of Actuaries on 25th November, 1867, and printed in abstract by order of the Council.]

IN this paper, Mr. Henry describes an instrument by the aid of which he has actually obtained the values (to four figures) of many thousand products, being the D values in the various tables he has published. The principle of the instrument, which he claims as